STSM Applicant: Juan Jose Salazar Gonzalez, Universidad de La Laguna (ES), jjsalaza@ull.es
STSM Topic: Designing efficient routes to serve customers with stochastic demands
Host: François Louveaux, University of Namur, Namur (BE), francois.louveaux@unamur.be

# Final report

Purpose of the STSM;

The purpose of the mobility was to progress and conclude a join research work that Salazar and Louveaux has initiated through emails. It is on a new model and algorithm for solving the capacitated vehicle routing with stochastic demands and with preventive returns. The "Capacitated Vehicle Routing Problem" (CVRP) is a classical problem in Combinatorial Optimization that involves assigning customers to vehicles and ordering the customers in a route for each vehicle to minimize the total travel cost. It has been approached by main researchers and a good survey is a recent book

P. Toth, D. Vigo. Vehicle Routing: Problems, Methods, and Applications. Society for Industrial and Applied Mathematics, 2014.

In the CVRP the customers are given with demands known before start planning. A more realistic variant is the so-called "CVRP with Stochastic Demands" (SDCVRP), where the customer demands are random variables. The works in the literature for this problem consider a penalty cost in the objective function for the cases where a vehicle fails to serve a customer in its route. See for example:

O. Jabali, W. Rei, M. Gendreau, G. Laporte. Partial-route inequalities for the multi-vehicle routing problem with stochastic demands. Discrete Applied Mathematics 177 (2014) 121-136.

C. Gauvin, G. Desaulniers, M. Gendreau. A branch-cut-and-price algorithm for the vehicle routing problem with stochastic demands. Computers & Operations Research 50 (2014) 141-153

We are now studying a father variant where we allow the vehicle to perform preventive returns, which is an action that reduces the probability of failures although in some cases could increase the travel cost. The problem is called "SDCVRP with preventive returns", and the only article addressing (heuristically) this problem is

W. Yang, K. Mathur, R.H. Ballou. Stochastic vehicle routing problem with restocking. Transportation Science 34 (2000) 99-112.

It is a quite difficult problem due to the complexity of modelling the expected penalty depending when the restocking action is done on each route. The intention of our research work is to publish the first article with an exact approach to solve the SDCVRP.

This is a join research work between Salazar and Louveaux. We are merging experience on developing branch-and-cut algorithms and studying stochastic problems, respectively. We started working on this problem around a year ago. We have collected and studied the literature already. We have also designed a mathematical formulation which is solvable through a cutting-plane technique similar to the L-shaped method

G. Laporte, F. Louveaux. The integer L-shaped method for stochastic integer programs with complete recourse. Operations Research Letters 13 (1993) 133-142.

We have also performed preliminary results based on our computer implementation using CPLEX as branch-and-cut framework. We have written the main results of our research, and the purpose of the STSM is to write a manuscript to be submitted to a scientific journal. Currently Salazar has not research funds in Spain to cover this trip. On the other hand, the topic of our work is highly related with the COST action TD1207 on energy production and distribution. Indeed, the vehicle routing problem is a logistic problem arising in many distribution networks, where product collected from a depot must be transported to customers. Our work is intended to contribute to support decision systems requiring a mathematical optimization tool to take better decisions when the customer demands are known in advance.

Description of the work carried out during the STSM;

We did achieve the purpose of the STSM. Indeed, we worked the 6 days of the mobility in writing an article that is now ready to be submitted to a Journal. It was not an easy task, and indeed we did a hard work during the 6 days to achieve the aim. We had to redesign some computational experiments, and indeed after the STSM new tables were inserted in the paper. We also worked hard to write a better introduction and motivation of the problem, together with updating the literature in the paper. Currently, thanks to the STSM, we now have a full manuscript with 22 pages that contains all the results of our research on the topic, and this manuscript is available to the public by request to the authors (although we also hope that it will published in a good scientific journal).

This manuscript considers a vehicle routing problem where the customer demands are stochastic variables. Due to uncertainty, along a route the vehicle may be unable to load all planned customers' demand. The vehicle has to return to the depot, unload and then resume its trip. In order to avoid unplanned return trips to the depot, one may decide to make some preventive return: even if it is not full, the vehicle returns to the depot, unload and resume its trip at the next customer. These preventive returns avoid visiting the same customer twice at the expense of possibly making an unneeded return. In this paper, we propose an exact procedure for designing routes to minimize the total expected cost of the routes. It consists of a branch-and-cut algorithm based on the L-shaped method for stochastic programs with binary first-stage variables. The paper provides lower bounds and cuts on the expected costs, and describe the results of computational analysis of a computer implementation on benchmark instances. Instances involving up to 100 customers have been solved to optimality.

Description of the main results obtained;

The main result of the STSM is a manuscript ready to be submitted to a top-level Journal on Transportation. The main novelty in the manuscript is a description of the first exact algorithm to solve the CVRP with stochastic demands and preventive returns. The algorithm is based on the L-shaped method, which follows a branch-and-cut approach based on an integer programming formulation. In this formulation has a non-linear objective function quite difficult to be evaluated since it requires computing the expected cost of solution. To deal with this non-linear term, we use an artificial variable representing a lower bound on the expected cost of a solution, and the performance of the approach depends strongly on the quality of this lower bound. Our paper describes several procedures to compute sophisticated lower bounds based on integer and fractional solutions. The lower bounds are then incorporated into the branch-and-cut framework, and we have designed separation approaches to find violated cuts. The approach was implemented on a computer and it succeeded solving benchmark instances with up to 100 customers. Our approach is the first finding optimal solution for these instances, and we do hope that future researchers can improve our results on this challenging problem.

Future collaboration with the host institution (if applicable);

We would like to go on merging our experiences on using the branch-and-cut framework for solving other vehicle routing problems with stochastic demands. We had considered a strategy that consists of applying preventive returns to the depot, but other strategies could also be considered. Therefore, there is space for future research, but currently we no specific plan on it. Our more immediate plan is to wait for the referee reports on our manuscript once it has been submitted to a good journal, and then go on based on such reports.

Universidad de La Laguna

Departamento de Matemáticas, Estadística e Investigación Operativa

<u>Foreseen publications/articles resulting from the STSM (if applicable);</u>

There will be an immediate publication as a result of the STSM, with the title:
F.V. Louveaux, J.J.Salazar-González. 2016.
"Exact approach for the vehicle routing problem with stochastic demands and preventive returns".

<u>Confirmation by the host institution of the successful execution of the STSM;</u>

The manuscript itself is a proof of the successful execution of the STSM.

<u>Other comments (if any).</u>

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Tenerife, 27th April 2016

Juan José Salazar González