



# CRC/Transregio 154

Mathematical Modelling, Simulation and Optimization  
using the Example of Gas Networks

Alexander Martin  
15.03.2015



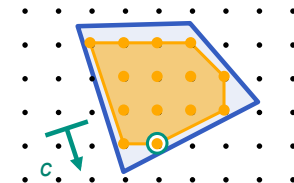
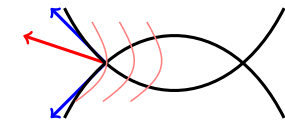
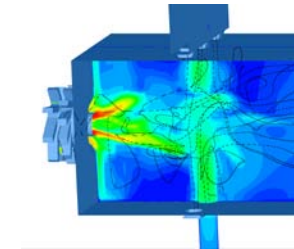
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Overview

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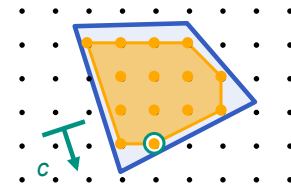
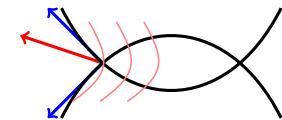
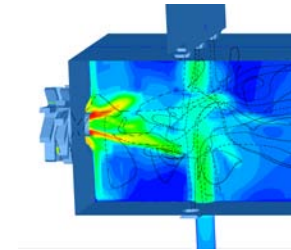
## Mathematical modeling, simulation, and optimization

- Modeling and numerical simulation
  - existence, uniqueness, regularity
  - efficient algorithms, convergence, error control
- Nonlinear optimization and control
  - efficient algorithms, convergence, error control
  - local optima and their characterization
- Integer programming
  - globally optimal
  - analysis of the solution space



## Mathematical modeling, simulation, and optimization

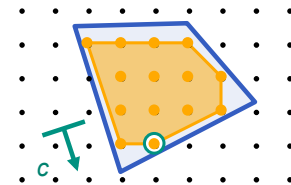
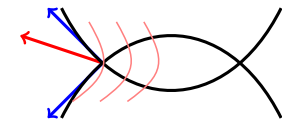
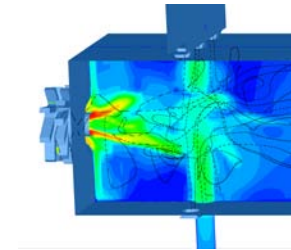
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### Coupling: integer and continuous

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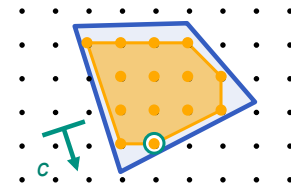
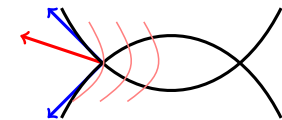
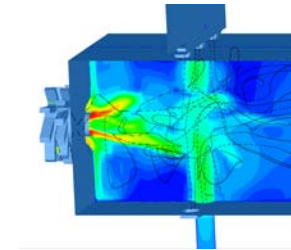


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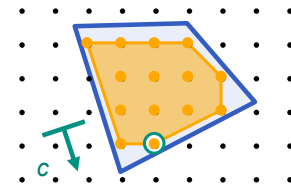
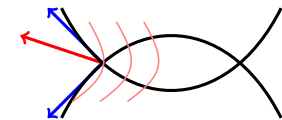
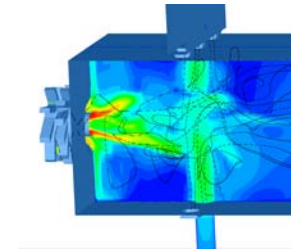


### Coupling: integer and continuous with uncertainty

- active research area
- uncertain data

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### Coupling: integer and continuous with uncertainty

- active research area
- uncertain data
- general methods out-of-reach

## Gas networks

- Networks are inherently discrete

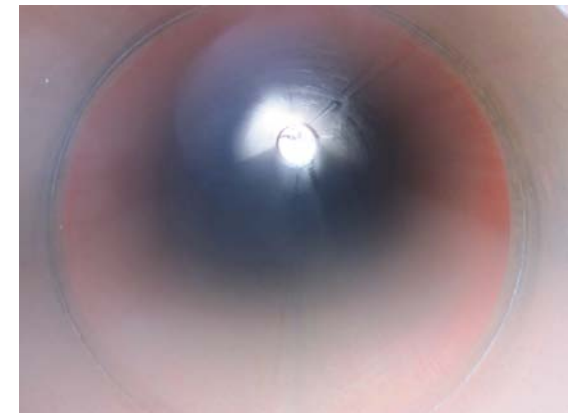
Ventil  $a$  with switching variable  $s_a \in \{0, 1\}$

$$s_a = 0 \Rightarrow q_a = 0$$

$$s_a = 1 \Rightarrow p_i = p_j$$

- Physics are inherently continuous

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} &= 0 \\ \frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho v^2 + p)}{\partial x} + g\rho \frac{\partial h}{\partial x} + \frac{\lambda}{2D}\rho |v| v &= 0 \\ \frac{\partial E}{\partial t} + \frac{\partial(Ev + pv)}{\partial x} + A\rho v g \frac{\partial h}{\partial x} + \pi D c_{HT}(T - T_{soil}) &= 0. \end{aligned}$$





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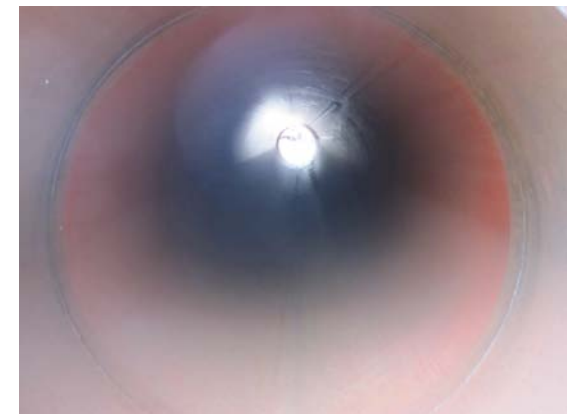
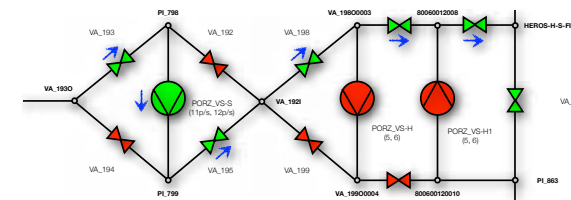
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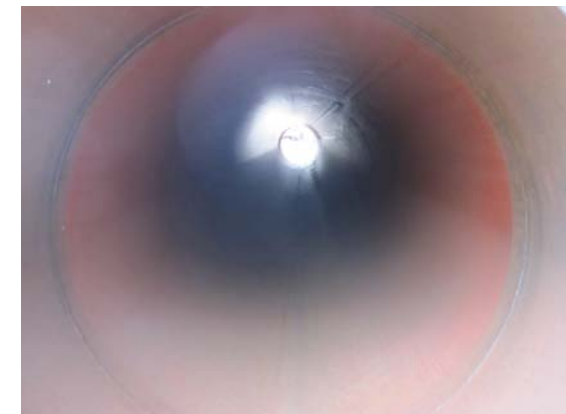
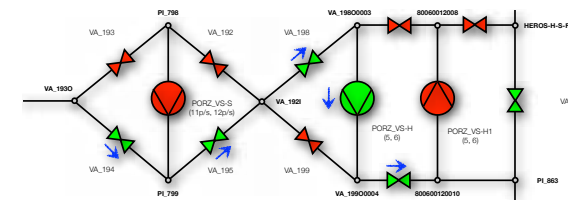
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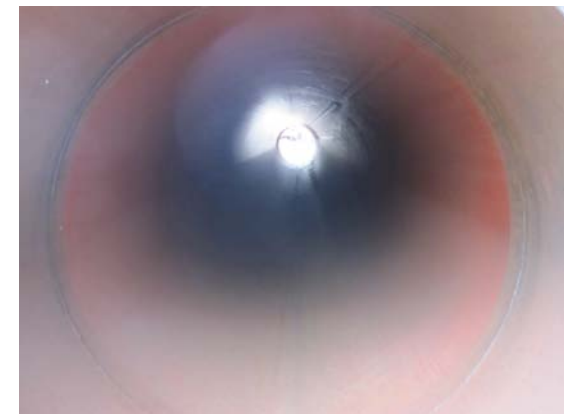
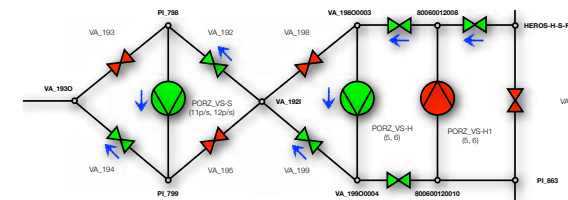
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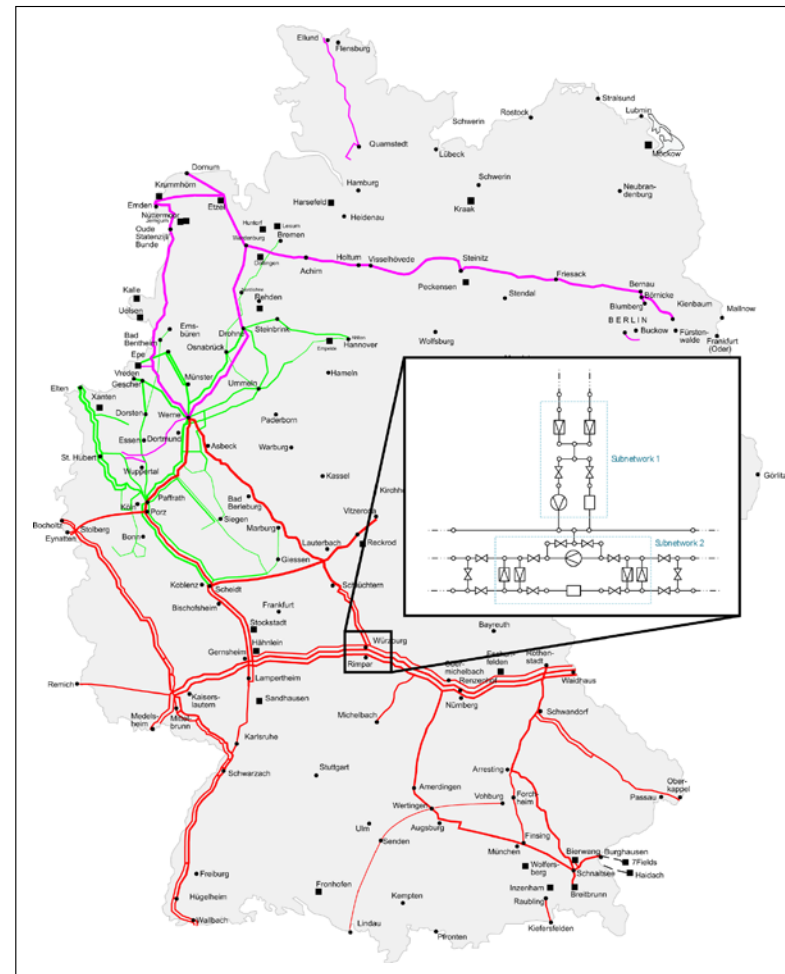
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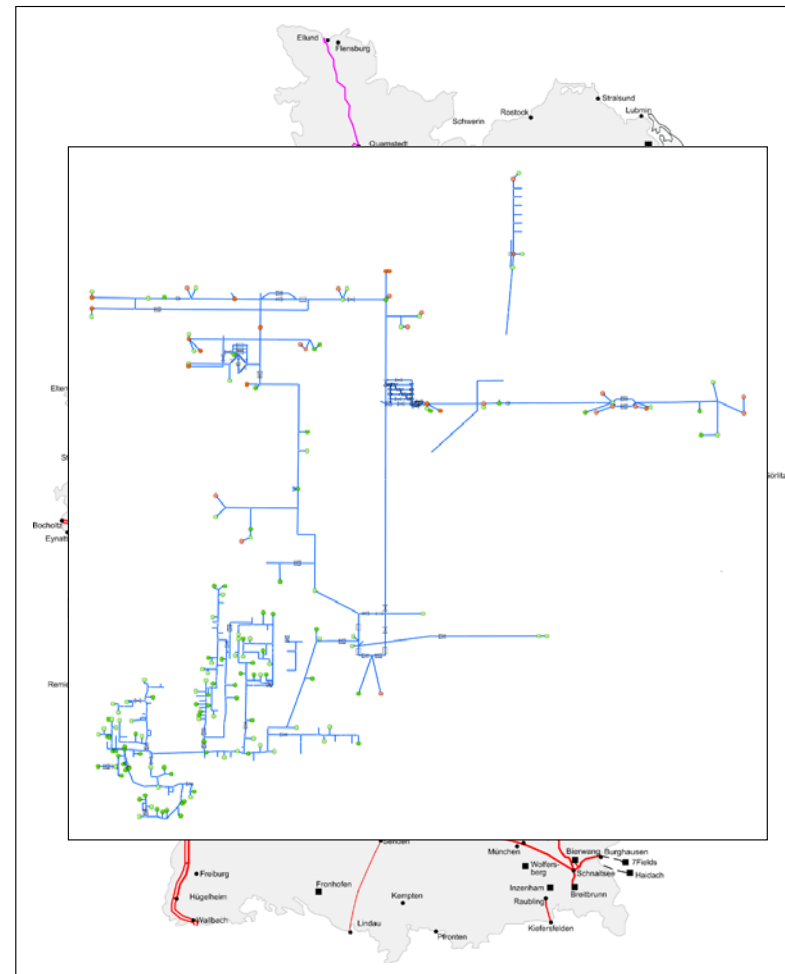
# Gas networks

- Gas networks are very big



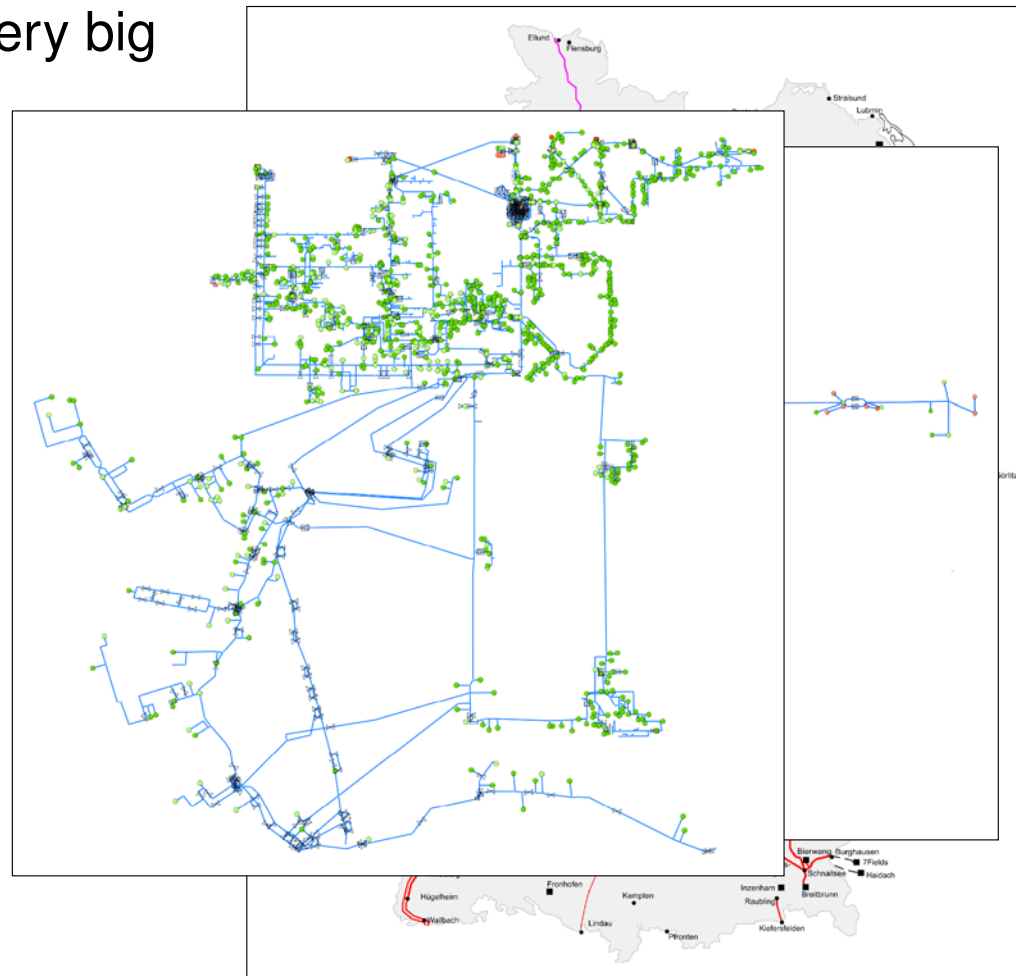
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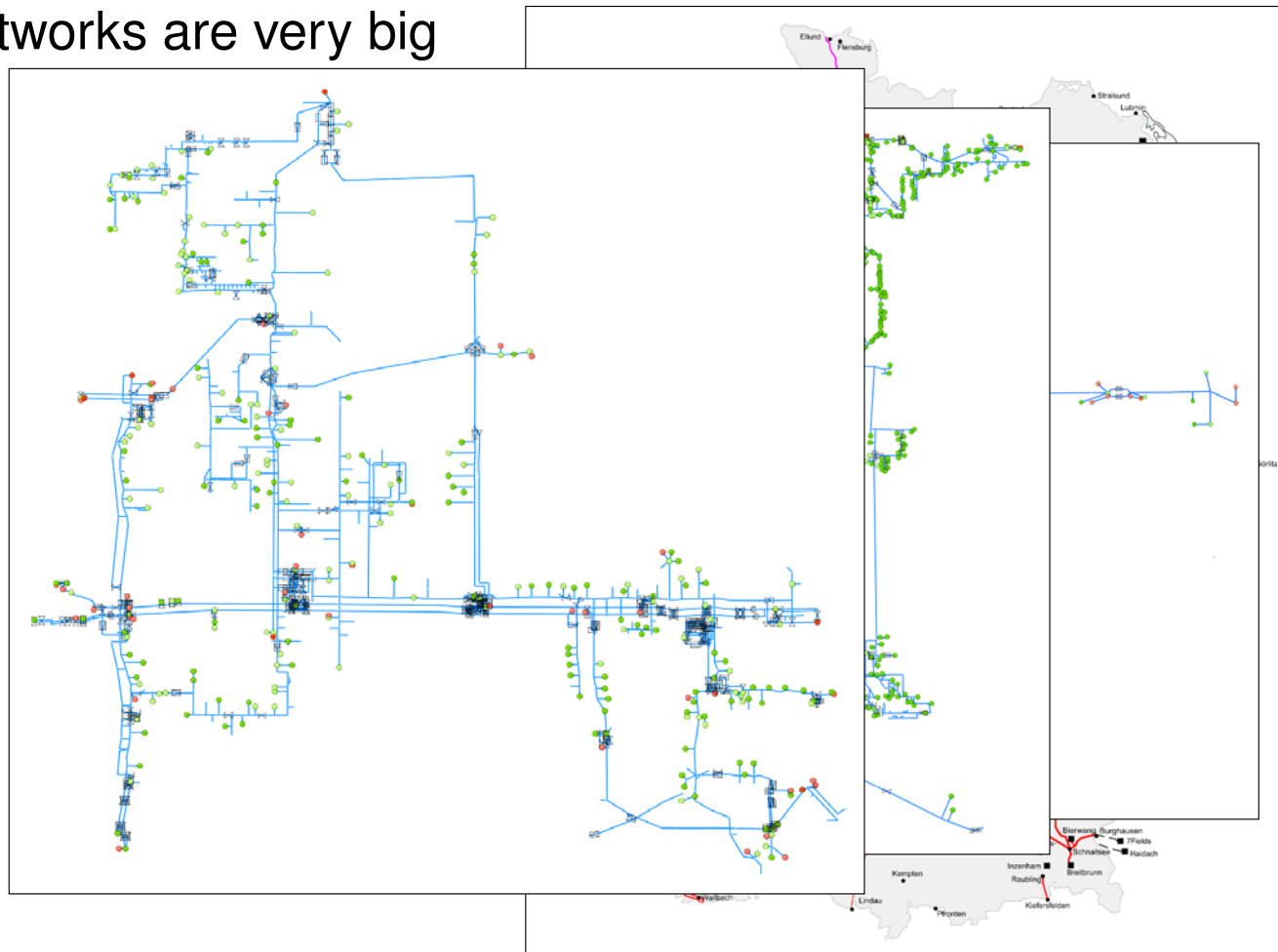
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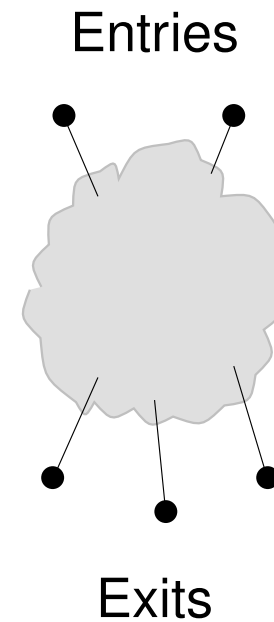
- Gas plays a major role in the transformation of the energy system
- Gas ...
  - is sufficiently available (within this time period)
  - is promptly disposable
  - is storable
  - is traded





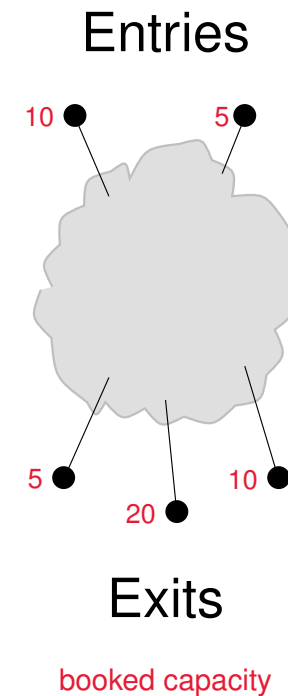
## Gas networks

- Exact methods are necessary
  - in optimization and
  - in simulation (via error estimators)



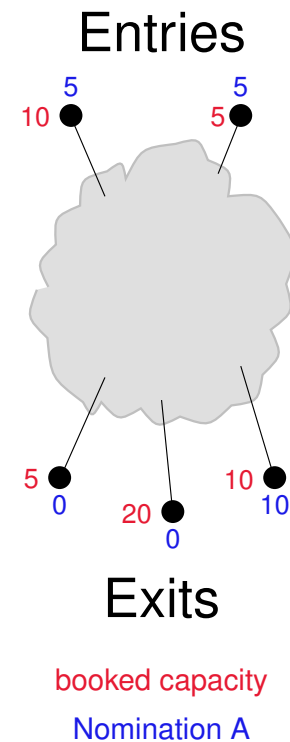
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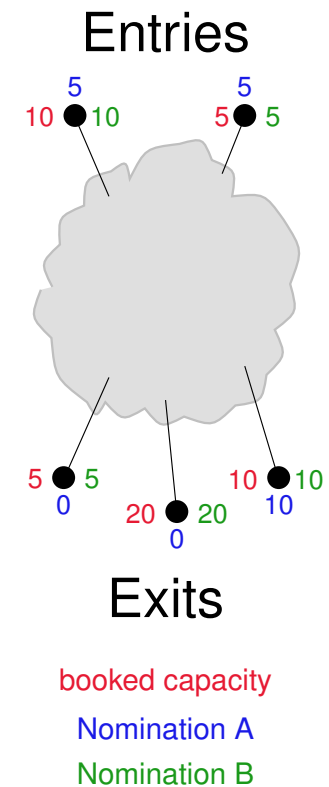
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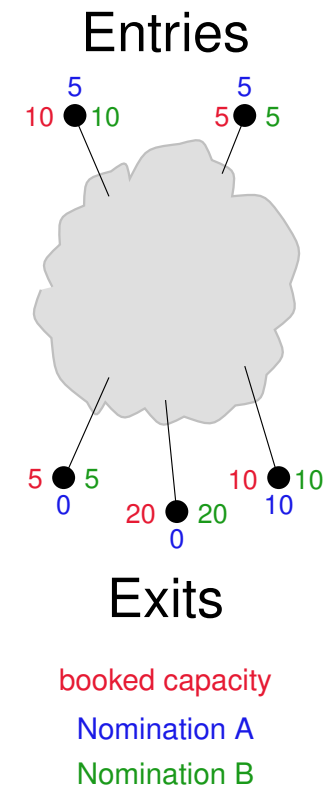
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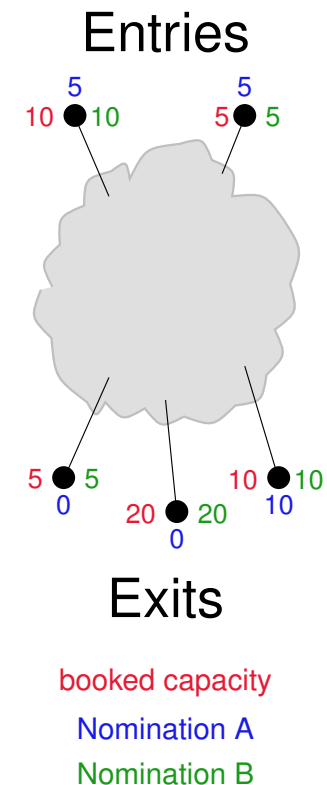
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- Distinction stationary/instationary is relevant
  - Market assumes stationary situations
  - Physics are instationary



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- Distinction stationary/instationary is relevant
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  - Physics are instationary
- Uncertainties
  - at the entry/exit nodes
  - in the model parameters



## Mathematical questions and goals using the example feasibility/infeasibility

In-/feasibility of nominations

- Consistent hierarchy of models and their coupling
- Error estimators and controls
- Exact methods and their coupling

**We need ...**

a global mathematical understanding of the input/output behaviour of optimally controlled dynamic networks

# Teamwork

- Graduate school



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- Integration in excellent existing structures  
**BMS**, **CE**, **ESE**, **EnCN**, ...

## Teamwork

- Graduate school
- Integration in excellent existing structures  
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- common data base

## Teamwork

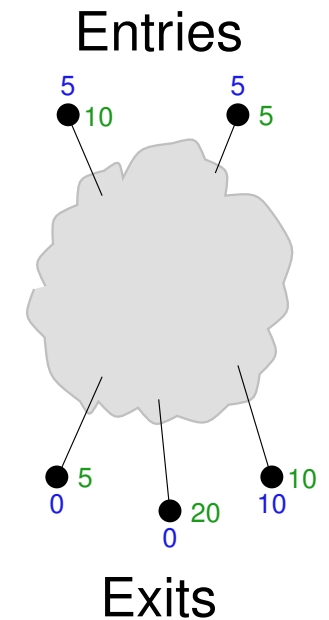
- Graduate school
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BMS, CE, ESE, EnCN, ...
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- Demonstrators

Demo	OPT MINLP	OPT ODE/PDE	SIM	Un- certainty	Model Coupling
Nomination	A07	A01, A05, B02	C02	B04, B05, C03	B03
Power Plant	A04	A02	C02, C04		A03
Storage	B07, A05	A02	B01, C04	B06	B01, B03

Berlin, Darmstadt, Erlangen, Duisburg

# Teamwork within Demonstrator 1

## How to get from Nomination A to Nomination B



Nomination A  
Nomination B

# Teamwork within Demonstrator 1

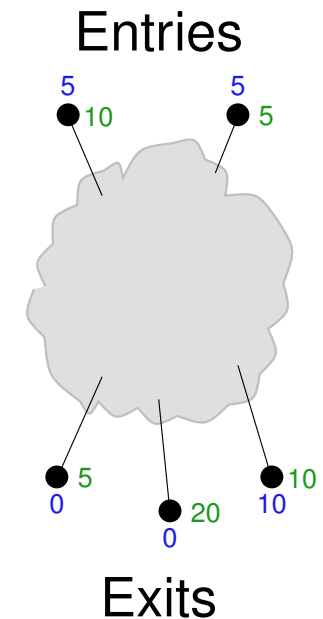
## How to get from Nomination A to Nomination B

stationary/*instationary*:

B07/07 Combinatorial decisions (on large networks)

A01/05 Corresponding optimal control

C02 Physical Validation



Nomination A  
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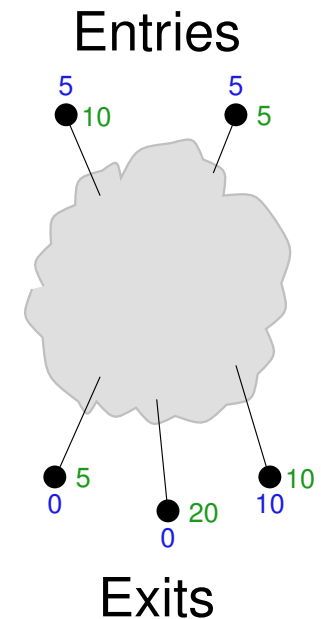
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Complemented by

C03 an analysis of reachability

B03 regulation of the solution interfaces



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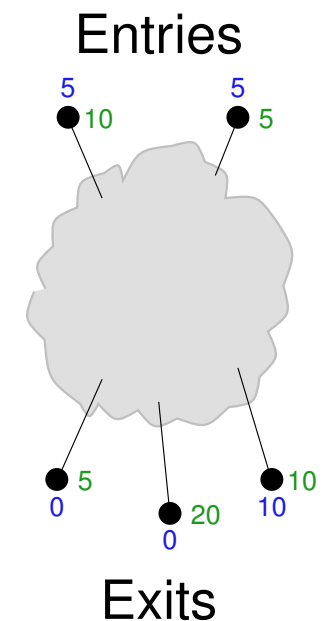
C03 an analysis of reachability

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Impact of uncertainties

B04/05 Existence of flows with chance constraints

B02 Robust Identification of parameters at small scale



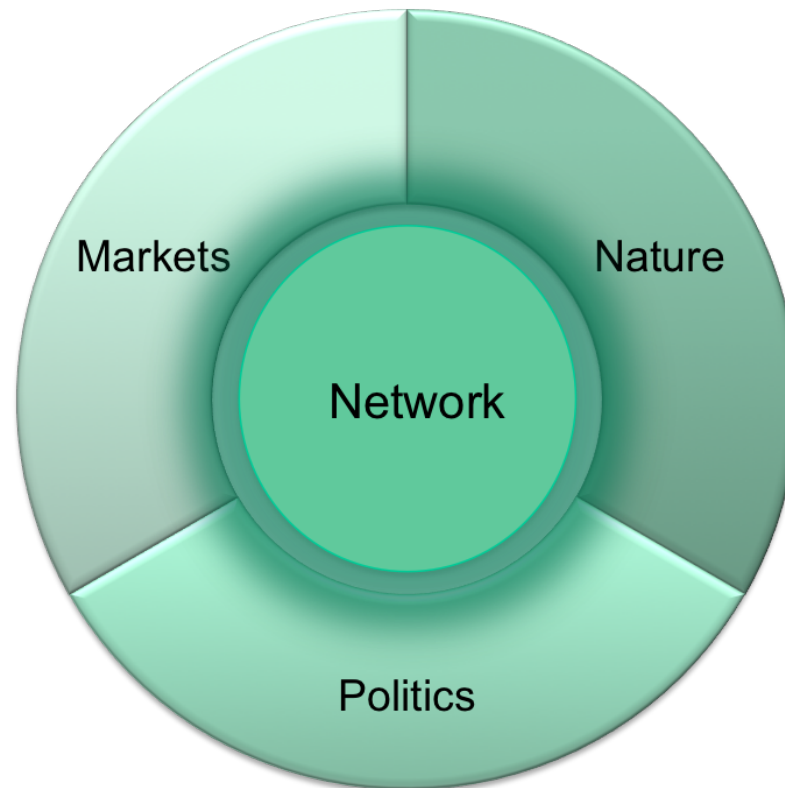
Nomination A  
Nomination B

## Topics across projects and sites

Topic	Berlin	Darmstadt	Erlangen	Essen
Discretization	C02	C04, B01	C02	
Uncertainty	B04		B06, C03	B05, C03
Switching Systems	A03	A02	B02	
Infeasibility	A07	A01	A05, B07	
Error Estimators / Control	B03, C02	B01	C02	
Model Coupling	A04, B03	B01	A03	



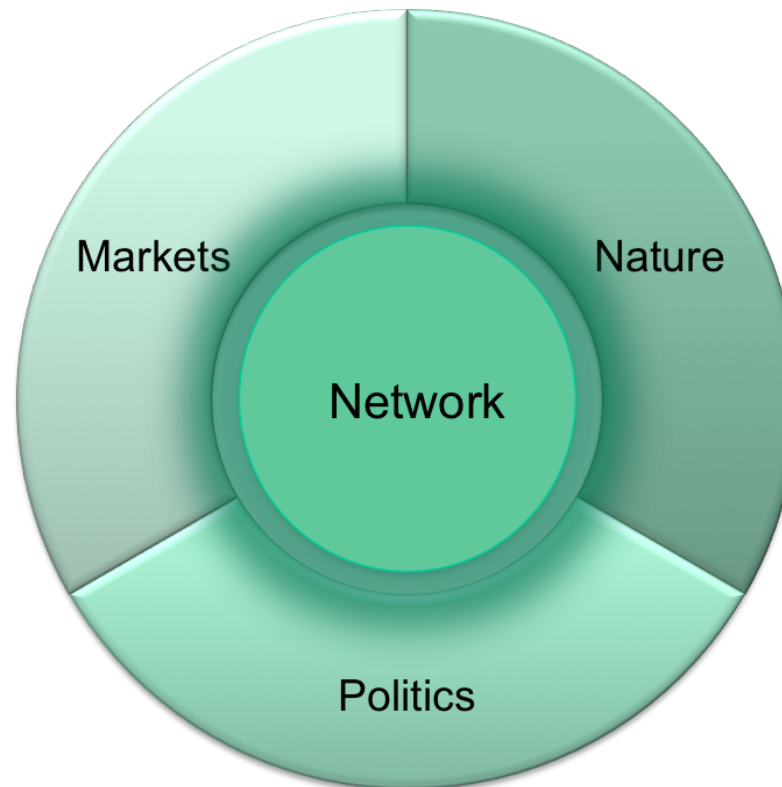
## TRR 154's future



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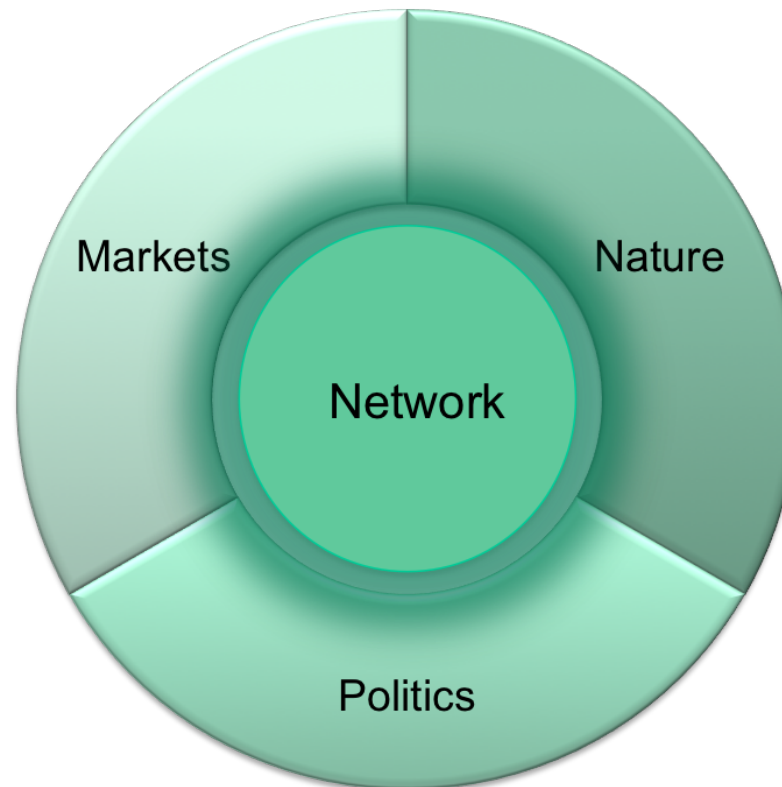
- Marketability
- Security of Supply
  
- Coupling of energy carriers



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### Gas networks

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### Mathematics

- Model extensions
- Increase in Complexity
- Multiscales and uncertainties
- Multilevel Optimization
- Coupling networks with physical transport