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Title: **Optimal ring topology for energy distribution's networks**

**Host:** Dr Tony Quek, SUTD, Singapore

**Applicant:** Dr Yoram Haddad, JCT, Israel (Early Stage Researcher)

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# Purpose of the STSM

The purpose of the STSM was first and foremost to discuss issues linked to energy distribution. For this purpose we would like to compare this problem with the somewhat equivalent one in the field of wireless networks. In addition, we wanted to discuss potential collaboration via bilateral program in our respective countries. Finally the applicant wanted to visit the Wireless Network and Decision Systems (WNDS) group led the host. This is described in the following sections.

# Description of the work carried out during the STSM and the main results obtained

It is a well-known fact that distribution energy network must be reliable to avoid failure and blackout. This requires a well thought topology. The choice of a network topology is mainly a matter of cost and availability consideration. There are several topologies that can be considered when we want to distribute the energy namely Tree or Ring topology. Since tree topology generally offers shorter paths and lower costs, while ring topology generally ensures a better availability, a ring–tree combination can be an efficient solution to cumulate the advantages of both technologies. In this STSM, we studied the following question: given an aggregation node (the production site or an important junction in the energy network) and n client sites, what is the best topology based on rings, each one of them including the aggregation node? This problem is similar to existing problem in Wireless Communications [1].

***Simplified model***

In a first step, we built a simplified model, based on the following five assumptions. Though the last two assumptions of this model are not realistic, this simplified approach will enable us to draw basic conclusions regarding backhaul network topologies.

Assumptions:

- the network includes $n$ sites (in addition to the aggregation node);
- the network topology is made of k rings;
- for $1 \leq i \leq k$ ring $i$ includes $n_i$ sites and the aggregation node; $n_1 \geq n_2 \geq \cdots \geq n_k \geq 2$;
- same failure probability for all links: $p$;
- failure events are uncorrelated.

$n$ and $n_i$ are related by the following equation:

$$n = \sum_{i=1}^{k} n_i \tag{1}$$

Availability: the condition for availability is that all sites be connected to the aggregation node. This condition is fulfilled if there is no more than one failure in each ring.

$$A = \prod_{i=1}^{k} ((1-p)^{n_i+1} + (n_i+1)p(1-p)^{n_i}) \tag{2}$$

If $p \ll 1$, this expression can be approximated by its second-order Taylor development:

$$A = \prod_{i=1}^{k} \left(1 - \frac{n_i(n_i+1)}{2} p^2 + o(p^2)\right)$$

$$A = 1 - p^2 \sum_{i=1}^{k} \frac{n_i(n_i+1)}{2} + o(p^2)$$

$$A = 1 - n\frac{p^2}{2} - \frac{p^2}{2} \sum_{i=1}^{k} {n_i}^2 + o(p^2) \tag{3}$$

In conclusion, increasing the number of rings reduces the maximum path length and unavailability. On the other hand, it requires more stations. In any case, given the number of rings, it is preferable that the variance of the ring size distribution be as small as possible and that the largest ring be as small as possible.

For a given number of rings $k$, the maximum availability is obtained when the number of sites are as close as possible to $\frac{n}{k}$. Let $q$ and $r$ be the quotient and the remainder of the Euclidean division of $n$ by $k$:

$$n = qk + r\,; \qquad 0 \leq r \leq k-1$$

Then, $n_1 = \cdots = n_r = q+1$ and $n_{r+1} = \cdots = n_k = q$

Therefore, the best availability is:

$$A_{k,p} = 1 - n\frac{p^2}{2} - \frac{p^2}{2}(r(q+1)^2 + (k-r)q^2) + o(p^2)$$

$$A_{k,p} = 1 - \frac{p^2}{2}(n + kq^2 + 2rq + r) + o(p^2) \qquad (4)$$

$A_{k,p}$ is an increasing function of $k$ and a decreasing function of $p$.

***General model***

We now assume that links may have various failure probability. The network includes one aggregation node ($O$) and $n$ sites $M_1, M_2, \dots, M_n$.

Let $p_i$ be the failure probability of the link $OM_i$ and $p_{ij}$ the failure probability of the link $M_iM_j$.

Assuming that each ring includes the aggregation node, a ring may be defined by an ordered sequence of sites.

Let define:

$$V = \{M_1, M_2, \dots, M_n\}$$

$R$: the set of rings including the aggregation node and sites of $V$.

For a ring $r \in R; r = (M_{i_1}, \dots, M_{i_m})$, the availability of $r$ is:

$$A(r) = (1-p_{i_1})(1-p_{i_m})\prod_{l=1}^{m-1}(1-p_{i_l i_{l+1}}) + p_{i_1}(1-p_{i_m})\prod_{l=1}^{m-1}(1-p_{i_l i_{l+1}}) + p_{i_m}(1-p_{i_1})\prod_{l=1}^{m-1}(1-p_{i_l i_{l+1}}) + (1-p_{i_1})(1-p_{i_m})\prod_{l=1}^{m-1}(1-p_{i_l i_{l+1}})\sum_{i=1}^{m-1}\frac{p_{i_l i_{l+1}}}{1-p_{i_l i_{l+1}}}$$

We try to maximize the following expression:

$$\max_k \max_{\substack{r_1 \cup \dots \cup r_k = E \\ r_i \cap r_j = \emptyset}} \prod_{i=1}^{k} A(r_i)$$

**Particular case: $\frac{n}{2}$ rings, each one including the aggregation node and 2 sites**

Here we assume the following:

- $n$ is an even number;

- the network includes $\frac{n}{2}$ rings, each one including the aggregation node and two sites;

- for $1 \le i \le n$, $p_i$ is the failure probability of the link $OM_i$;

- for $1 \le i, j \le n$, $p_{ij}$ the failure probability of the link $M_iM_j$.

We can calculate the availability of the ring $OM_iM_j$.

$$A_{ij} = (1-p_i)(1-p_j)(1-p_{ij}) + p_i(1-p_j)(1-p_{ij}) + p_j(1-p_i)(1-p_{ij}) + p_{ij}(1-p_i)(1-p_j)$$

$$A_{ij} = 1 - p_i p_j(1-p_{ij}) - p_i p_{ij}(1-p_j) - p_j p_{ij}(1-p_i) - p_i p_j p_{ij}$$

$$A_{ij} = 1 - p_i p_j - p_i p_{ij} - p_j p_{ij} + 2p_i p_j p_{ij}$$

We try to maximize the expression:

$$\prod_{(O,M_i,M_j)\in R} A_{ij}$$

The problem can be regarded as a search of a perfect matching in a weighted graph: given $G = (V, E, c)$ an undirected weighted graph, the goal is to compute a perfect matching (ie a subset of edges E'⊆E such that each node in V has exactly one incident edge in E') for a minimum cost $c(E')$.

The maximum 2-ring division problem can be solved efficiently (in polynomial time) as followed: given a network as described earlier, an undirected weighted graph G = (V, E) should be constructed where $V = \{M_1, M_2, \dots, M_n\}$ and $E = \left\{(M_i, M_j) \mid i, j \in \binom{n}{2}\right\}$ (a full graph). The weight function $W: E \to \mathbb{R}$ is defined as followed: $\forall\, i, j \in \binom{n}{2},\ w(M_i, M_j) = \log A_{i,j}$. Then, due to (3), finding a maximum 2-ring division in the original network is equivalent to finding a matching M in G such that for each matching M' in G, $\sum_{(M_i,M_j)\in M} w(M_i, M_j) \geq \sum_{(M_i,M_j)\in M'} w(M_i, M_j)$.

This problem is a well-known problem called maximum weighted matching. In 1964, Jack Edmonds was the first to develop a polynomial time algorithm to solve this problem [2]. A strait forward implementation of Edmonds' algorithm will have a running time complexity of $O(|V|^2|E|)$, and hence in our problem - $O(|V|^4)$ (because the constructed graph is fully meshed. i.e. $E = \theta(|V|^2)$). Over the years, several variants, implementations and improvements of Edmonds' idea where suggested, some of them in [3-5]. Overall, the best know algorithm for a full graph has a running time complexity of $O(|V|^3)$ [4-5], [8].

Now, solving the maximum 2-ring division problem is done in 2 phases:

1. Computing $\log A_{i,j}$ for each $i, j \in \binom{n}{2}$ and constructing an undirected weighted full graph G, as described earlier.
2. Solving the weighted maximum matching problem on G.

The running time complexity of phase 1 is $\binom{n}{2} * \theta(1) + \theta(n^2) = \theta(n^2)$. The running time complexity of phase 2 is $O(n^3)$. Therefore, the running time complexity of the proposed algorithm for solving the maximum 2-ring division problem is $O(n^3)$. Hence, the decision problem corresponding to the maximum 2-ring division problem is in P.

<u>Conclusion:</u> it is possible to connect an even number $n$ of sites with $\frac{n}{2}$ rings, each of one including 2 sites and the aggregation node. The running time is $O(n^3)$.

**[1] Ron Nadiv and Tzika Naveh. Wireless Backhaul Topologies: Analysing Backhaul Topologies Strategies, Ceragon White Paper, August 2010.**

**[2] Edmonds, Jack. "Maximum matching and a polyhedron with 0, l-vertices."** ***J. Res. Nat. Bur. Standards B*** **69.1965 (1965): 125-130.**

**[3] Gabow, Harold N. "Implementation of algorithms for maximum matching on nonbipartite graphs." (1974).**

**[4] Gabow, Harold N. "An efficient implementation of Edmonds' algorithm for maximum matching on graphs."** ***Journal of the ACM (JACM)*** **23.2 (1976): 221-234.**

**[5] Gabow, Harold N.** ***Data structures for weighted matching and nearest common ancestors with linking*****. University of Colorado, Boulder, Department of Computer Science, 1990.**

**[6] Kolmogorov, Vladimir. "Blossom V: a new implementation of a minimum cost perfect matching algorithm."** ***Mathematical Programming Computation*** **1.1 (2009): 43-67.**

**[7] Elad Hazan, Shmuel Safra, and Oded Schwartz. "On the complexity of approximating k-set packing."** ***computational complexity*** **15.1 (2006): 20-39.**

**[8] Schrijver, Alexander.** ***Theory of linear and integer programming*****. John Wiley & Sons, 1998. p 227.**

## Future collaboration with the host institution

The next step would be to discuss the more general case where General case: n/k rings, each one including the aggregation node and k sites; k≥3 which will require to investigate the correlation between the general maximum k-ring division problem and a NP-Complete problem.

In addition we are considering to apply for a bilateral research grant between our both countries soon.

## Confirmation by the host of the successful execution of the STSM

Attached letter.