Scientific Report about the STMS

In the first part of Doc-course, we were followed many courses in "Optimization and Applied Mathematics", which was very rich and very interesting. This has allowed us to learn some information and technics about numerical methods (finite element method, finite volume method), and how to use some programs (Fenix, Matlab, Python, ...) to solve and investigate our numerical problems.

On the other hand, we studied how to investigate an optimization problem and the methods used to find and calculate the optimal minimum.

Moreover, there was some course about modeling and real problems, and how to solve these problems using the optimization and numerical methods.

In the second part, I worked with professor Enrique Fernández-Cara on the topic of Control Theory, and we have considered the optimal control problem of 1D linear Schrödinger equation:

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\begin{align*}
    iy_t - y_{xx} + V(x)y &= 0 \quad (x, t) \in (0, 1) \times (0, T), \\
    y(0, t) &= y(1, t) = 0 \quad t \in (0, T), \\
    y(x, 0) &= y_0(x) \quad x \in (0, 1),
\end{align*}
\]

- \( y = y(x, t) \) is the complex-valued state function.
- \( V(x) \in W^{1,\infty}((0, 1); \mathbb{R}) \) is the control.
- \( y_0(x) \in H^1_0(0, 1) \),

then, we showed that problem (1) has at least one optimal control \( \hat{V} \in U_{ad} := W^{1,\infty}(\mathbb{R}^N) \cap \{ V : |V(x)| + |V'(x)| \leq M \} \). Furthermore, we introduced some algorithms (Fixed point iterates, Gradient descent iterates, ...) to compute our optimal control \( \hat{V} \). (Please, see the Presentation attached to the mail )