Scientific Report on the STSM
"Numerically Robust Integer-Feasibility"

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1 Motivation

Mixed-integer linear programming and its extensions towards nonlinearities are a key technology for the optimization of production and distribution of energy and other utilities, see Carrión and Arroyo [2006], Fügenschuh et al. [2013], and Gleixner et al. [2012a] for only a few examples of recent publications on this topic. State-of-the-art tools for mixed-integer programming use limited-precision arithmetic and are hence prone to numerical inaccuracies.

While in practice for the majority of applications this is perfectly satisfactory, there are applications where high-precision or exact solutions are required. In the energy sector, one example are feasibility problems occuring in the fair distribution of network capacities among traders, see, for instance Pfetsch et al. [2014].

2 Goal

The goal of this short term scientific mission was to investigate methods to efficiently test whether the *integer* assignments returned by floating-point MIP solvers are reliable. This is interesting, because for many applications these determine the implementation of a solution in practice. To this end we will use recent advances in the exact and accurate solution of linear programs as developed by Gleixner et al. [2012b] and integrate them into the state-of-the-art MIP and MINLP solver SCIP [Achterberg, 2007].

3 Mission Report

The mission was conducted at the Operations Research Group at the Università degli Studi di Bologna. Within the local OR seminar, the grantee presented his latest research on solving linear programs exactly over the rational numbers. Building on this, the main host Andrea Lodi, his student Luca Fabbri, and the grantee developed an experimental setup designed to systematically investigate the reliability of integer assignments produced by floating-point MIP solvers. Subsequently, the grantee worked on an implementation to link the MIP solver SCIP with the LP solver SoPlex such as to be able to conduct the experiment. This is described in more detail in the next section.

4 Results

The first outcome of the mission was a concise definition of "reliability of integer assignments" in MIP solvers that is amenable to computational analysis. For this, suppose a floating-point MIP solver produces an approximately feasible solution vector $x^*, y^* \in \mathbb{R}^{n+m}$ during the course of its branch-and-bound search, where the variables in x should be integer and the variables in y may be continuous. We consider the linear program obtained by rounding the values in x^* and fixing the integer variables to these rounded values. This LP can be solved exactly over the rational numbers.

Depending on the result, three cases can be distinguished:

- the LP is (exactly) feasible and the objective function value is identical to the one of the floating-point solution
- the LP is (exactly) feasible and the objective function value differs from the one of the floating-point solution
- the LP is infeasible in exact arithmetic

The last case is the strongest form of unreliability, but depending on the magnitude of the deviation already the second point is undesirable. If the floating-point solution is *superoptimal* this can lead to incorrect pruning during the subsequent search. If the floating-point solution is *suboptimal* this can lead to a incorrect (suboptimal) solution value reported by the MIP solver and affect its performance because of an unnecessarily large search tree.

As a second result, an implementation within the SCIP solver [Achterberg, 2007] has been started that can use the exact solving capabilites of the SoPlex LP solver by Gleixner et al. [2012b] in order to perform the above analysis numerically. This analysis will lead to a clearer picture both on the price of precision within MIP solving, but also on the benefits that can be gained by exact computation.

The main part of the implementation consists of a constraint handler for the SCIP framework. This will be made publically available within one of the next releases of the SCIP Optimization Suite and can then be accessed and used by the academic community.

5 Collaboration and Publications

As a final outcome, this short term scientific mission helped to intensify the collaboration between the Department of Optimization at Zuse Institute Berlin and the host institution at the University of Bologna. The results of this ongoing research will be described in a common publication targeted at a computationally oriented journal for optimization and operations research. We hope to finalize this submission until autumn 2015.

References

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