Report on COST TD1207 STSM

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Abstract

We report the main activities and results performed when the first author was visiting the second author during COST TD1207 STSM.

1 Purpose of the STSM

In real-time prediction of coastal bay conditions - such as water quality and circulation patterns - for environmental decision support, the underlying fluid dynamic model is often described by the Navier-Stokes equations. Projections of Navier-Stokes equations yield Optimal Control Problems (OCP) with polynomial data, a notoriously hard problem to solve.

The purpose of the stay was two-fold:

1. Develop practical solutions to solve the particular projection problem.
2. Develop efficient moment relaxations to obtain global solutions of structured polynomial optimal control problems.

It was thought that objective 2 would be necessary for 1, but as outlined in the next sections, progress was achieved on both fronts in a relatively independent fashion.

2 Description of the work carried out during the STSM and the main results obtained

Concerning objective 1, it was decided to first develop a local method to solve the OCP, for comparison with the global method. First of all, an easier optimization problem was identified, from which dynamics could be integrated, leading to a finite-dimensional optimization problem. This problem could then be solved by standard local (gradient-descent) algorithms. It was identified that this approach performed surprisingly well, leading to near-global solutions. This hints that the problem has a convex structure, and the tools of global optimization are therefore unnecessary. Then, the OCP was approached head-on, using a direct method in optimal control implemented through the BOCOP software [1]. On the example at hand, the method also performed well.

Concerning objective 2, a more fundamental research activity was initiated, since there was no need to develop a specific global method for the particular problem. The research focused on a recent method for polynomial optimization, developed by Bissan Ghaddar of the host institution. The method [2] proposes a Dynamic Inequality Generation Scheme (DIGS) for those problems, which does not require a hierarchy of ever-increasing semi-definite relaxations such as Lasserre's approach [3]. Therefore, problems of a much higher size can be handle. The method, however, does not converge in all cases. The collaboration identified a crucial modification for the algorithm, and the improved algorithm was extensively benchmarked. On a theoretical side, the convergence of the new algorithm was identified to be related to an open question in moment problems concerning the extension of positive semi-definite matrices to moment matrices of positive measures. See attached file for more details.

3 Future collaboration with the host institution

Concerning objective 1, the methods developed have been tested on a benchmark problem with a simplified data set as a first step during the stay. As a next step, they need to be applied to a test case with realistic data. Further collaboration is therefore expected.

Concerning objective 2, the algorithm needs to be extended and benchmarked for structured static optimization problems and optimal control problems. Both numerical experiments to prove efficiency and theoretical work to prove convergence of the scheme are needed, which would be a major result in the field.

4 Foreseen publications/articles resulting from the STSM

Actions pertaining objective 1 are expected to lead to an application paper, after validation on a realistic model.

Further research for the DIGS algorithm is necessary before publication.

5 Confirmation by the host institution of the successful execution of the STSM

See attached files.

References

[1] J. Bonnans, Frédéric, Pierre Martinon, and Vincent Grélard. Bocop - A collection of examples. Research Report RR-8053, INRIA, 2012.

[2] B. Ghaddar, J. C. Vera, and M. F. Anjos. A dynamic inequality generation scheme for polynomial programming. 2013.

[3] Jean Bernard Lasserre. Optimisation globale et théorie des moments. *Comptes Rendus de l'Académie des Sciences - Series I - Mathematics*, 331(11):929 – 934, 2000.