Scientific Report

STSM on Bi-objective Mixed Integer Non-Linear Programming

In this document we describe the work carried out during the short-term scientific mission from the 21st to the 31st of October 2013 of Dr. Valentina Cacchiani to LIX, Ecole Politechnique. The research was done together with Dr. Claudia D'Ambrosio.

Purpose of the STSM and description of the work carried our during the STSM

The purpose of the STSM has been to study Bi-objective Mixed Integer Non-Linear Programming (MINLP) and its application to energy production.

We have examined the existing literature in the field of Multi-objective Mixed Integer Linear Programming (MILP) problems and Multi-objective Non-Linear Programming (NLP) problems, and in particular, we have focused on the following works:

M. J. Alves, J. Clımaco. A review of interactive methods for multiobjective integer and mixed-integer programming, European Journal of Operational Research 180 (2007) 99–115.

P. Belotti, B. Soylu, M.M. Wiecek. A branch-and-bound algorithm for biobjective Mixed-integer Programs, Techincal Report *www.optimization-online.org/DB_FILE/2013/01/3719.pdf*

M. Ehrgott, L. Shao, A. Schöbel. An approximation algorithm for convex multi-objective programming problems, J Glob Optim 50 (2011) 397–416.

J. Fernández, B. Tóth. Obtaining an outer approximation of the efficient set of nonlinear biobjective problems, J Glob Optim 38 (2007) 315–331.

S. Leyffer. A Complementarity Constraint Formulation of Convex Multiobjective Optimization Problems, INFORMS Journal on Computing 21 (2) (2009) 257–267.

G. Mavrotas, D. Diakoulaki. A branch and bound algorithm for mixed zero-one multiple objective linear programming, European Journal of Operational Research 107 (1998) 530-541.

G. Mavrotas, D. Diakoulaki. Multi-criteria branch and bound: A vector maximization algorithm for Mixed 0-1 Multiple Objective Linear Programming, Applied Mathematics and Computation 171 (2005) 53–71.

They include two exact branch and bound methods for MILP problems and approximated methods for convex non-linear multi-objective problems. Two commonly used methods for deriving an approximated Pareto set consist of *weighted sum* and *goal* (also known as e-constraint).

By examining the existing literature on the topic, we have observed that there isn't any exact method on multi-objective convex MINLP problems. Our goal has been to study a general framework for this kind of problems. In particular, we have focused on the general structure of the branch-and-bound algorithm by Mavrotas and Diakoulaki and have studied how to extend it to deal with non-linear convex problems.

We have decided to focus on an application for energy production, studied in A. Borghetti, C. D'Ambrosio, A. Lodi, S. Martello. A MILP Approach for Short-Term Hydro Scheduling and Unit Commitment with Head-Dependent Reservoir, IEEE Transactions on Power Systems 23 (3), pp. 1115–1124, 2008. The studied problem consists of a unit commitment problem whose aim is to find the optimal scheduling of a multi-unit pump-storage hydro power station for a given time horizon of one week. In that work, a single objective function was considered aiming at maximizing the total profit obtained by energy production. The objective function was a complex non-linear function which was linearized. One of the constraints was to fix the total volume of the reservoire at the end of the considered time horizon to a fixed amount.

We have considered a simplified version of the problem, which includes a shorter time horizon (one day) and simplifies the head effects on power production. We have extended the simplified model by considering two objectives, i.e. the maximization of the total profit and the maximization of the final total volume. In addition, we have approximated the non-linear function by a convex (quadratic) function.

We have first applied the two methodologies of weighted sum and goal to the convex multi-objective problem. We have shown that even with a time horizon of one hour and a single turbine, the Pareto set presents discontinuities. This is a characteristic of multi-objective mixed-integer (linear and non-linear) problems, which makes impossible to obtain all the Pareto points by weighted sum or goal methods. Finally, we have started the development of the branch-and-bound algorithm. Both methods have been developed in AMPL.

Description of the main results obtained

We have learned existing techniques on multi-objective problems and applied them to an application in energy production. The results obtained confirm the expectations by showing an approximated Pareto set with solutions having a trade-off between profit and final volume obtained.

We have shown that, for our application, even with a time horizon of one hour and a single turbine, the Pareto set presents discontinuities.

We have designed a general branch-and-bound algorithm for multi-objective convex MINLP problems, by extending existing methods for multi-objective MILP problems.

Future collaboration and foreseen publications

The collaboration will continue with the goal of developing the proposed branch-and-bound algorithm and applying it to the energy production application. We also plan to prepare a report and submit it to an international journal.

Confirmation by the host institution of the successful execution of the STSM

See the attached document.